



M2 Internship Proposal

Iterative HDG finite element method for dissipative wave propagation problems

Location:	ENSTA, Paris-Saclay Campus (Palaiseau), France
Laboratory:	POEMS (CNRS, Inria, ENSTA) in collaboration with LAUM (CNRS, Le Mans U)
Supervisors:	Axel Modave (CNRS, POEMS) and Gwénaél Gabard (Le Mans U., LAUM)
Duration:	5 to 6 months, academic year 2026–2027
Fields:	Computational Mechanics, Applied Mathematics
Keywords:	Finite element methods, Hybridizable discontinuous Galerkin, Iterative solvers, Time-harmonic waves, Dissipative/porous media, MATLAB

Scientific Context

Dissipative media play a key role in the aerospace, automotive and energy sectors, for comfort and performance as well as for health and safety (e.g. porous materials for cabin noise reduction or vibration damping devices in buildings). Additive manufacturing now makes it possible to design these materials at the microstructural scale, but optimizing them requires computing the acoustic response of a large number of samples, typically by solving many time-harmonic wave propagation problems. Effective models relying on physical and geometric approximations are commonly used because they are inexpensive, but this comes at the price of a loss of accuracy and reliability that is not always clearly identified or quantified. Fully resolved models, solved with **finite element methods (FEMs)**, are well suited to complex geometries and physics, but are costly in terms of memory and computing time.

Accelerating FEM solvers for time-harmonic wave propagation problems is a key challenge in applied mathematics and computational mechanics. Several strategies are being investigated, such as domain decomposition methods (DDMs), adaptive FEMs, model order reduction, as well as non-conforming methods, including the hybridizable discontinuous Galerkin (HDG) finite element method. In the HDG method, an auxiliary unknown is introduced on the mesh skeleton, and the physical unknowns are eliminated element by element, resulting in a global system posed on the skeleton that is better suited to iterative solvers.

We recently introduced the **CHDG method**, which combines ideas from DDMs and HDG methods. The main unknown is a transmission variable defined on the mesh skeleton [1]. This formulation yields fast-converging iterative solvers with an algorithmic structure well suited to parallel computing [2]. So far, CHDG has only been applied to non-dissipative problems [3], and its extension to dissipative media is the motivation for this internship.

Objective and Methodology

The main objective of this internship is to **extend the CHDG method to wave propagation problems in dissipative media**. The project combines a literature review, the design and analysis of numerical methods, and numerical experiments. It will be organized in four steps:

1. **State of the art.** Review existing DG and HDG methods for dissipative wave propagation problems.
2. **Design.** Propose and compare several strategies to extend the CHDG method to dissipative homogeneous media, building in particular on the approach of [4].
3. **Analysis.** Extend the convergence proofs obtained in the non-dissipative case [1, 3] to the new formulations.
4. **Numerical study.** Implement the formulations in an existing 2D MATLAB code¹ and assess their accuracy, as well as the convergence of the iterative solvers, on 2D benchmarks.

The intern will work in close collaboration with researchers from POEMS and LAUM, and will gain experience in both the mathematical analysis and the implementation of advanced finite element methods.

Candidate Profile

This internship is intended for a student in the final year of an engineering school and/or a Master's program (M2) in computational mechanics or applied mathematics.

- Solid background in partial differential equations and finite element methods;
- Experience in scientific programming (MATLAB; C++ is a plus);
- Interest in both numerical analysis and numerical experiments;
- Autonomy, rigor, and good communication skills in English or French.

This internship could lead to a PhD thesis, although no funding is secured at this stage.

Application

Send a CV and your most recent academic transcripts to Axel Modave (axel.modave@ensta.fr) and Gwénaél Gabard (gwenael.gabard@univ-lemans.fr).

References

- [1] A. Modave and T. Chaumont-Frelet. A hybridizable discontinuous Galerkin method with characteristic variables for Helmholtz problems. *Journal of Computational Physics*, 493:112459, 2023.
- [2] A. Modave, R. Greffe, C. Geuzaine, A. Rappaport, and A. Chabib. Méthode d'éléments finis discontinus hybridisée pour la résolution itérative accélérée de problèmes d'ondes en fréquence. In *17ème Colloque National en Calcul des Structures*, 2026.
- [3] S. Pescuma, G. Gabard, T. Chaumont-Frelet, and A. Modave. A HDG method with transmission variables for time-harmonic wave propagation problems with constant coefficients. *arXiv preprint arXiv:2606.14950*, 2026.
- [4] G. Gabard and O. Dazel. A discontinuous Galerkin method with plane waves for sound-absorbing materials. *International Journal for Numerical Methods in Engineering*, 104(12):1115–1138, 2015.

¹WavesDGLab, <https://github.com/wavesdglab/wavesdglab>